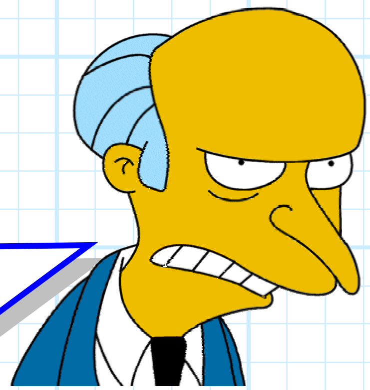
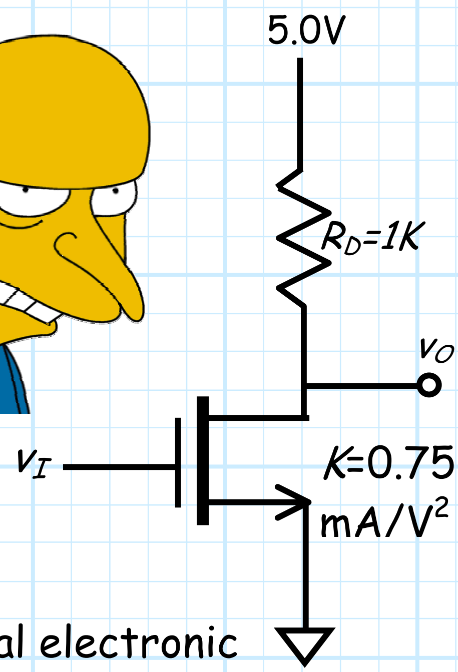
The MOSFET as an Amp and Switch

Consider this **simple** MOSFET circuit:

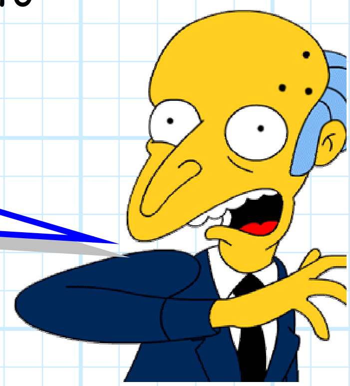
Q: *Oh, goody—you're going to **waste** my time with another of these **pointless** academic problems. Why can't you discuss a circuit that actually **does** something?*



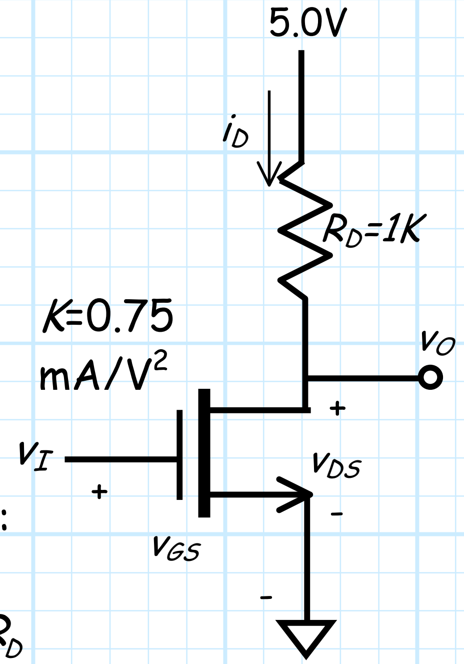


A: Actually, this circuit **is** a fundamental electronic device! To see what this circuit does, we need to determine its **transfer function** $v_O = f(v_I)$.

Q: ***Transfer function!*** *How can we determine the transfer function of a MOSFET circuit!?*



A: **Same** as with junction diodes—we determine the output v_O for each device mode, and then determine **when** (i.e., for what values of v_I) the device is in that mode!



First, note that **regardless** of the MOSFET mode:

$$v_{GS} = v_I - 0.0 = v_I$$

and:

$$v_{DS} = v_O - 0.0 = v_O$$

From KVL, we can likewise conclude that:

$$v_{DS} = v_O = 5.0 - i_D R_D$$

Now let's ASSUME that the MOSFET is in **cutoff**, thus ENFORCING i_D=0.

Therefore:

$$\begin{aligned} v_O &= 5.0 - i_D R_D \\ &= 5.0 - 0(1) \\ &= 5.0 \text{ V} \end{aligned}$$

Now, we know that MOSFET is in cutoff **when**:

$$v_{GS} = v_I < V_t = 1.0$$

Thus, we conclude that:

$$\boxed{v_O = 5.0 \text{ V} \quad \text{when} \quad v_I < 1.0 \text{ V}}$$

Now, let's ASSUME that the MOSFET is in **saturation**, thus ENFORCE:

$$\begin{aligned} i_D &= K\left(v_{GS} - V_t\right)^2 \\ &= K\left(v_I - V_t\right)^2 \\ &= 0.75\left(v_I - 1.0\right)^2 \end{aligned}$$

And thus the output voltage is:

$$\begin{aligned} v_O &= 5.0 - i_D R_D \\ &= 5.0 - 0.75\left(v_I - 1.0\right)^2 (1) \\ &= 5.0 - 0.75\left(v_I - 1.0\right)^2 \end{aligned}$$

Now, we know that MOSFET is in saturation **when**:

$$v_{GS} = v_I > V_t = 1.0$$

and when:

$$v_{DS} = v_O > v_{GS} - V_t = v_I - 1.0$$

This second inequality means:

$$v_O > v_I - 1.0$$

$$5.0 - 0.75\left(v_I - 1.0\right)^2 > v_I - 1.0$$

$$0 > 0.75\left(v_I - 1.0\right)^2 + \left(v_I - 1.0\right) - 5.0$$

Solving this quadratic, we find that the **only** consistent solution is:

$$v_I - 1.0 < 2.0$$

$$v_I < 3.0$$

Meaning that the MOSFET is in saturation when $v_I > 1.0$ and $v_I < 3.0$. Logically, this is same thing as saying the MOSFET is in saturation when $1.0 < v_I < 3.0$.

Thus we conclude:

$$\boxed{v_O = 5.0 - 0.75\left(v_I - 1.0\right)^2 \quad \text{when} \quad 1.0 < v_I < 3.0 \ \text{V}}$$

Finally, let's ASSUME that the MOSFET is in **triode** mode, thus we ENFORCE:

$$\begin{aligned} i_D &= K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right] \\ &= 0.75\left[2\left(v_I - 1.0\right)v_O - v_O^2\right] \end{aligned}$$

And thus the output voltage is:

$$\begin{aligned} v_O &= 5.0 - i_D R_D \\ &= 5.0 - 0.75\left[2\left(v_I - 1.0\right)v_O - v_O^2\right](1) \\ &= 5.0 - 0.75\left[2\left(v_I - 1.0\right)v_O - v_O^2\right] \end{aligned}$$

Rearranging this equation, we get the quadratic form:

$$0.75\, v_O^2 - \left(1.5\, v_I - 0.5\right)v_O + 5.0 = 0$$

The solutions of which are:

$$v_O = \frac{\left(1.5 v_I - 0.5\right) \pm \sqrt{\left(1.5 v_I - 0.5\right)^2 - 15.0}}{1.5}$$

Note because of the $\pm$, there are **two** possible solutions. However, to be in triode region, the MOSFET must not be in pinchoff, i.e.:

$$v_O = v_{DS} < v_{GS} - V_t = v_I - 1.0$$

This condition is satisfied with the **smaller** of the two solutions (i.e., the solution with the minus sign!):

$$v_O = \frac{(1.5v_I - 0.5) - \sqrt{(1.5v_I - 0.5)^2 - 15.0}}{1.5}$$

So, the above expression provides us with the output voltage **if** the MOSFET is in triode mode. The question remaining is thus **when** (i.e., for what values of V_I) is the MOSFET in triode mode?

We could do a lot more math to find this answer, but this answer is actually quite **obvious**!

Recall that we have already determined that:

a) The MOSFET is in cutoff **when** $v_I < 1.0$ V

b) The MOSFET is in saturation **when** $1.0 < v_I < 3.0$ V

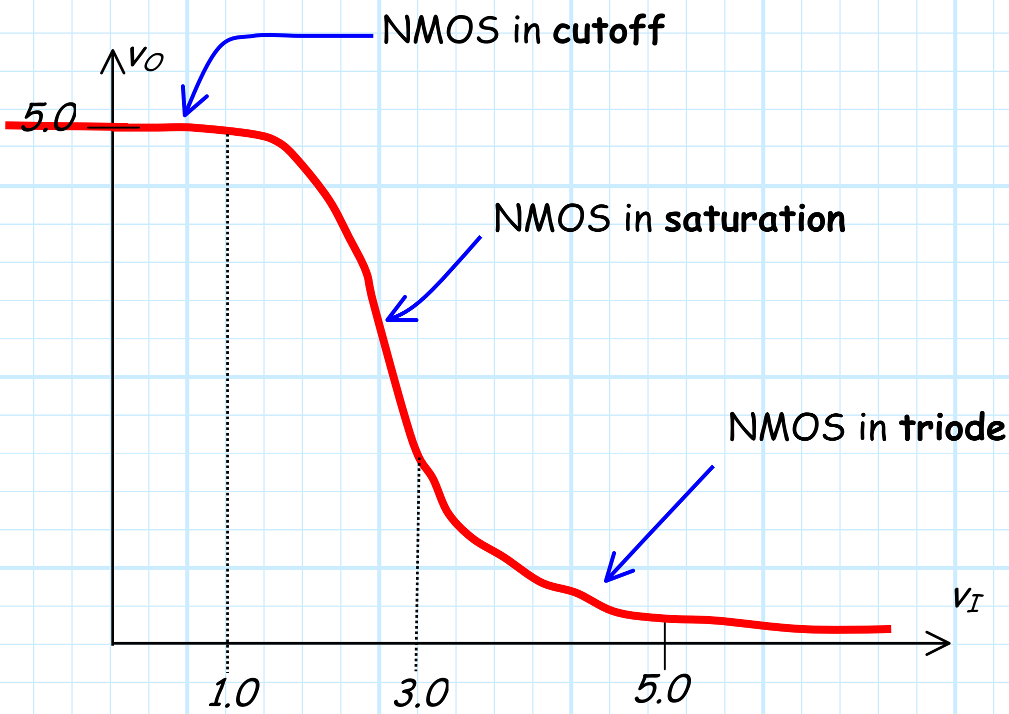
Since there are only **three** modes of a MOSFET device, and since the transfer **function** must—well—be a **function**, we can conclude (correctly) that the MOSFET will be in triode region when v_I is the value of the **only region that is left**--$v_I > 3.0$!

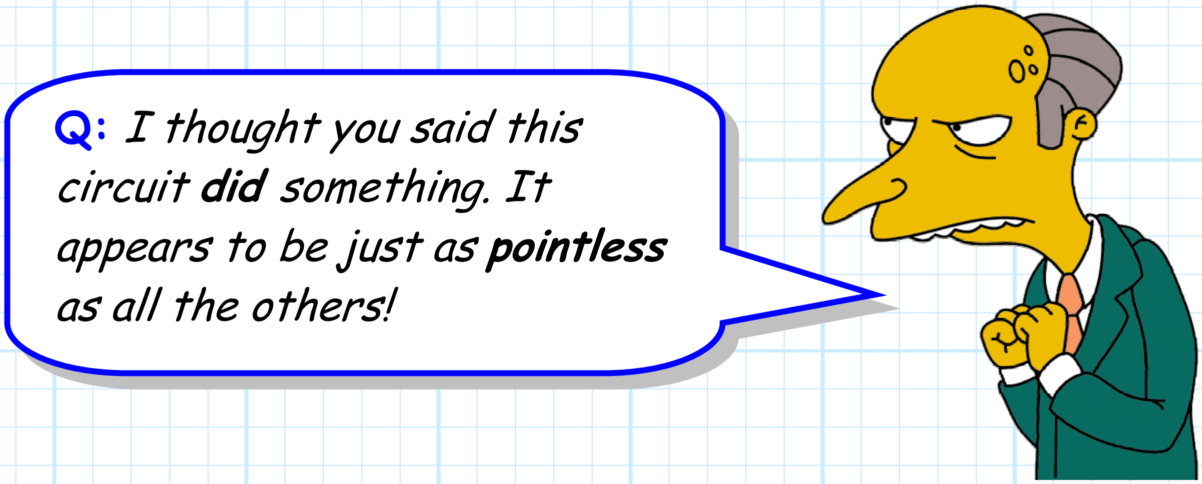
Thus we can conclude that:

$$v_O = \frac{(1.5v_I - 0.5) - \sqrt{(1.5v_I - 0.5)^2 - 15.0}}{1.5} \quad \text{when} \quad v_I > 3.0 \text{ V}$$

We now have determined the complete, continuous **transfer function** of this circuit!

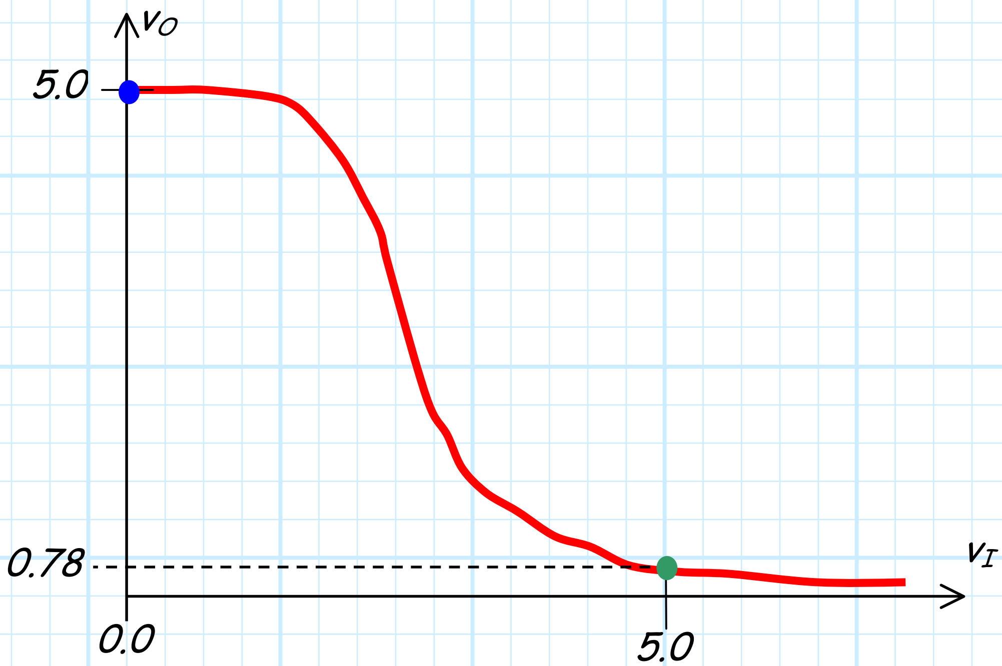
$$v_O = \begin{cases} 0 & \text{when} \quad v_I < 1.0 \text{ V} \\ 5.0 - 0.75(v_I - 1.0)^2 & \text{when} \quad 1.0 < v_I < 3.0 \text{ V} \\ \dfrac{(1.5v_I - 0.5) - \sqrt{(1.5v_I - 0.5)^2 - 15.0}}{1.5} & \text{when} \quad v_I > 3.0 \text{ V} \end{cases}$$





A: To see how this circuit is **useful**, consider what happens when the **input** voltage v_I is 0 V and 5V.

From the transfer function, we find that **if** $v_I = 0$, the output voltage will be $v_O = 5.0$. Likewise, **if** the input voltage is $v_I = 5.0$, the output voltage will be **small**, specifically $v_O = 0.78 \text{ V}$.



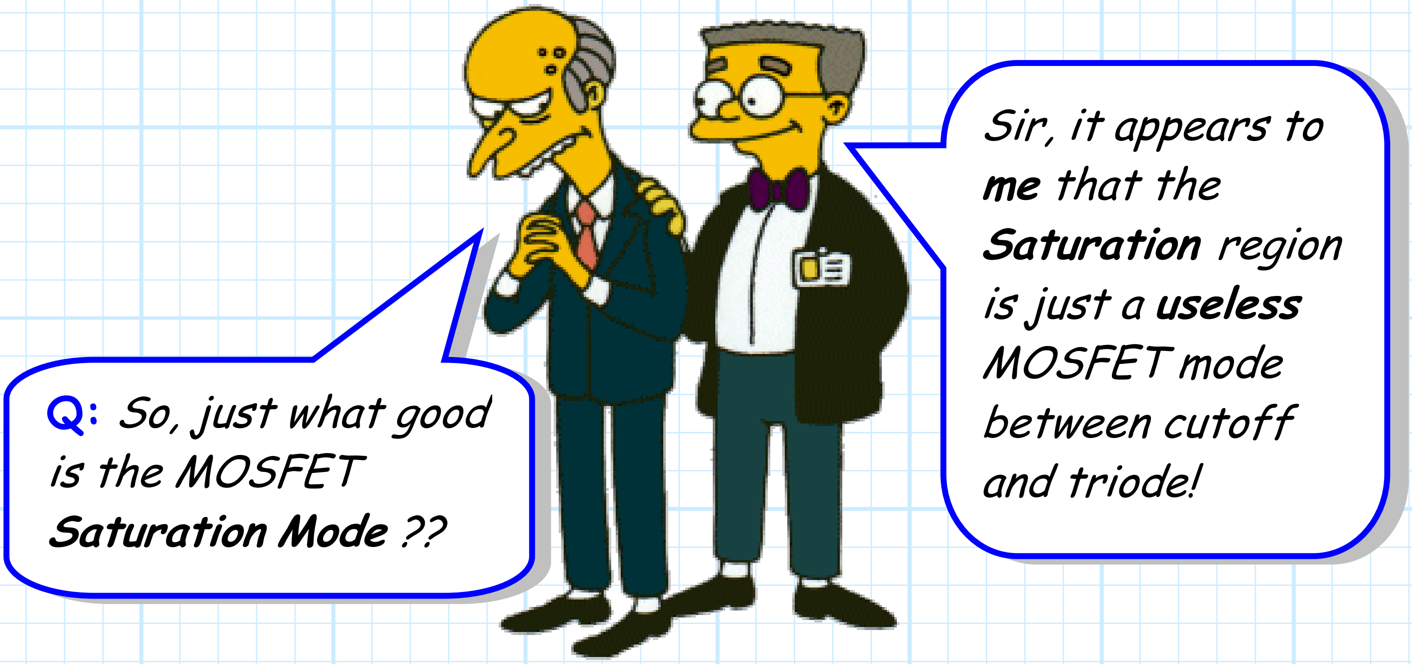
Let's **summarize** these results in a table:

v_I	v_O	Mode
0.0	5.0	Cutoff
5.0	small (0.78)	Triode

This circuit provides a **simple** example of **one** of the primary **applications** of MOSFET devices—**digital circuit design**. We can use MOSFETs to make digital devices such as **logic gates** (AND, OR, NOR, etc.), **flip-flops**, and digital **memory**.

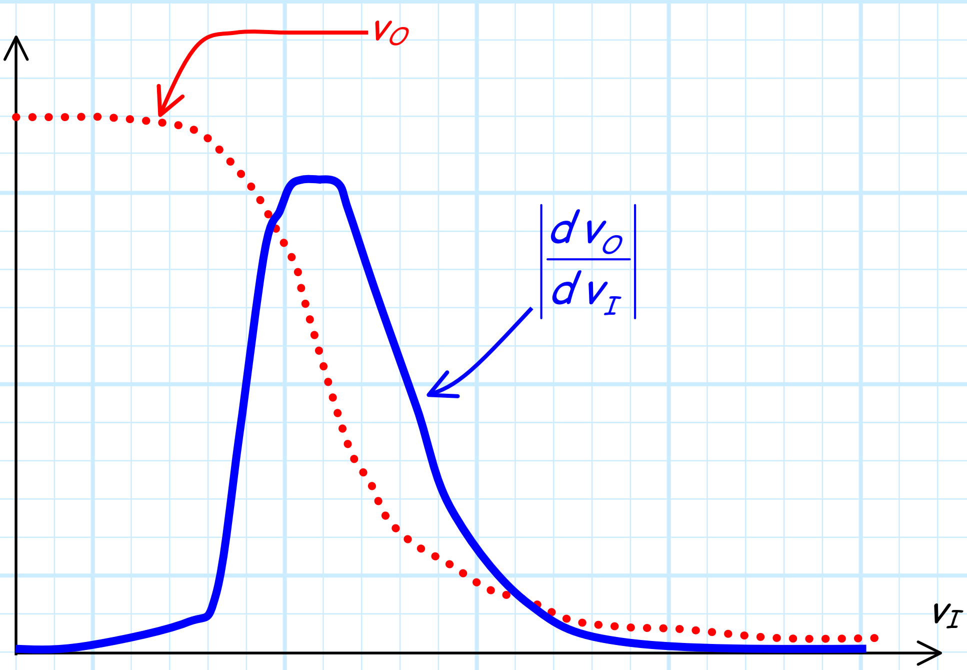
We typically find that, just like this circuit, when a MOSFET digital circuit is in **either** of its **two** binary states (i.e., "0" or "1"), the MOSFETs in the circuit will either be in **cutoff** (i_D=0) or in **triode** (v_{DS} small) modes.

→ **Cutoff** and **Triode** are the MOSFET modes associated with **digital** circuits and applications!



A: Actually, we will find the MOSFET **saturation** mode to be **extremely useful**!

To see why, take the **derivative** of the above circuit's transfer function (i.e., dv_O/dv_I):



We note that in **cutoff** and **triode**:

$$\left| \frac{dV_O}{dV_I} \right| \approx 0$$

while in the **saturation** mode:

$$\left| \frac{dV_O}{dV_I} \right| >> 1$$

Q: *Oh goody. The **slope** of the transfer function is **large** when the MOSFET is in **saturation**. Am I supposed to be impressed by that?! How are these results even **remotely** important!?*

A: Since in **cutoff** and **triode** $dv_O / dv_I = 0$, a **small change** in input voltage v_I will result in almost **no change** in output voltage v_O.

Contrast this with the **saturation** region, where $\left| dv_O / dv_I \right| >> 1$. This means that a **small change** in input voltage v_I results in a **large change** in the **output** voltage v_O!

To see how this is important, consider the case where the input signal has both a **DC** and a **small-signal** (AC) component:

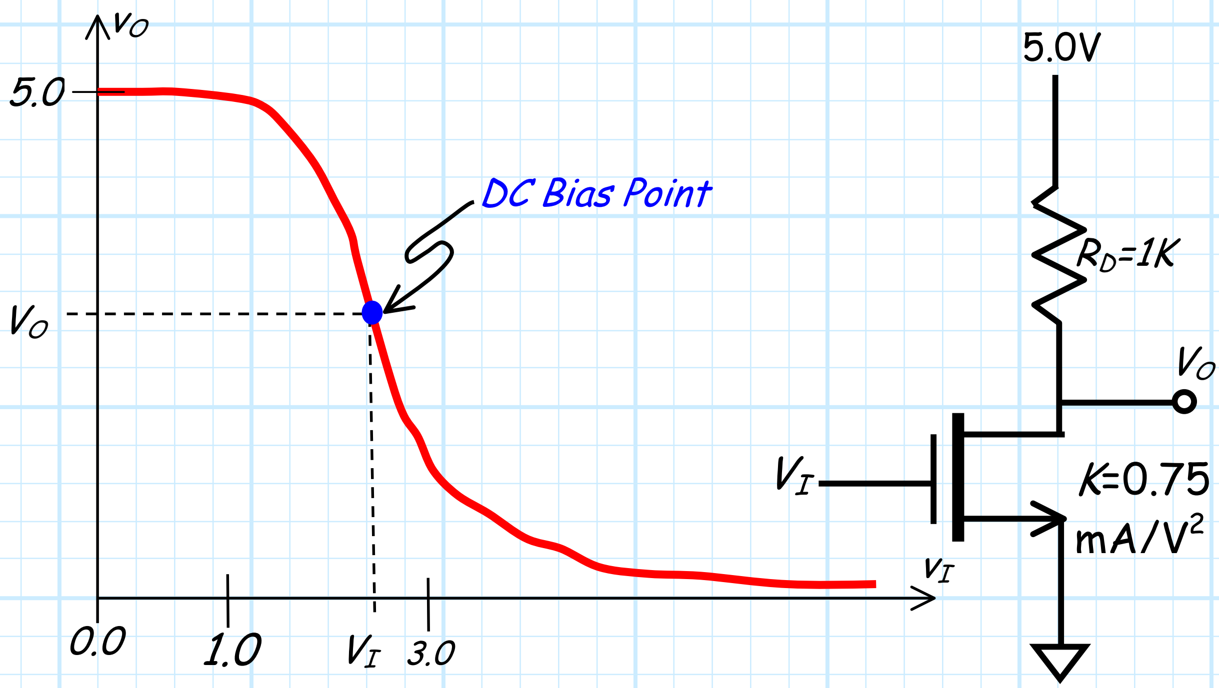
$$v_I(t) = V_I + v_i(t)$$

As a result, the **output** voltage likewise has both a DC and small-signal component:

$$v_O(t) = V_O + v_o(t)$$

Now, let's consider **only** the **DC** components. We can select the DC input V_I such that the MOSFET is placed in **saturation**. The value V_I, along with the resulting DC output V_O, sets a **DC bias point** for this circuit.

By selecting the right value of V_I we **could** set this DC bias point to where the transfer function **slope** is the greatest:

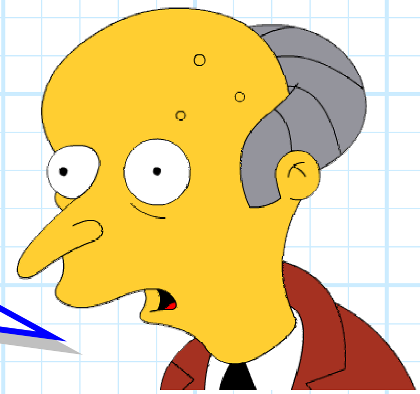


Now, say we **add** a **small-signal** v_i to this input DC voltage (i.e., $v_I(t) = V_I + v_i(t)$). This small signal simply represents a small change in the input voltage from its **average** (i.e., DC) value. The result is of course as **small change** in the **output** voltage—the **small-signal output voltage** $v_o(t)$!

Now for the **interesting** part (I bet you were wondering when I would get around to it)! The small change in the output voltage will have a much **larger** magnitude than the small change in the input!

For example, if the input voltage changes by **1 mV** (i.e., v_i =1mV), the output **might** change by, say, **5 mV** (i.e., v_o = 5 mV).

Q: *Goodness! By how much would the **output** change in our example circuit? How can we **determine** the small-signal **output** v_o ??*



Determining how much the output voltage of our circuit will change when we change the input voltage by a small amount is very **straightforward**—we simply take the **derivative** of the output voltage v_O with respect to input voltage v_I!

By taking the **derivative** of v_O with respect to v_I (when the MOSFET is in saturation, we find:

$$\frac{d\,v_O}{dv_I} = \frac{d\left(5.0 - 0.75\left(v_I - 1.0\right)^2\right)}{dv_I}$$

$$= -1.50\left(v_I - 1.0\right) \qquad \text{for} \quad 1.0 < v_I < 3.0$$

This expression describes the **slope** of our circuit's transfer function (for 1.0 < v_I < 3.0). Note the slope with the **largest magnitude** occurs when v_I=3.0, providing a **slope** of -3.0 mV/mV.

Thus, **if** we DC bias this circuit with V_I = 3.0 V (resulting in V_O = 2.0 V), we find that the small signal output will be **3 times** the small signal input!

For example, say that the **input** to our circuit is:

$$v_I = 3.0 + 0.01 \cos \omega t \quad \text{V}$$

(i.e., $V_I = 3.0$ V and $v_i = 0.01 \cos \omega t$). We would find that the **output voltage** would approximately be:

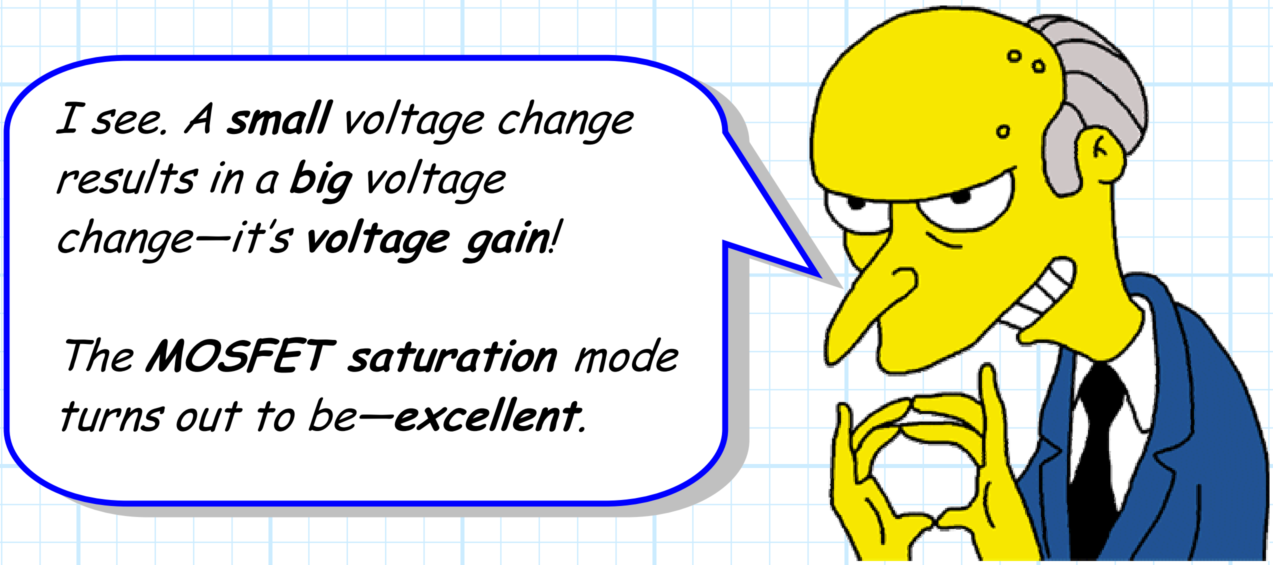
$$v_O = 2.0 - 0.03 \cos \omega t \quad \text{V}$$

(i.e., $V_I = 2.0$ V and $v_o = -0.03 \cos \omega t$). Note then that:

$$\begin{aligned} v_o &= \left. \frac{d\, v_O}{d v_I} \right|_{v_I = 3.0} v_i \\ &= -3.0\, v_i \\ &= -0.03 \cos \omega t \end{aligned}$$

In other words, the magnitude of the small-signal output has a magnitude **three times larger** than the input magnitude.

We say then that our signal provides **small-signal gain**—our circuit is also a **small-signal amplifier**!



Even the simple circuit of this example is sufficient demonstrates to demonstrate the **two primary applications** of MOSFET transistors--**digital** circuits and signal **amplification**.

Whereas the important MOSFET regions for **digital** devices are **triode** and **cutoff**, MOSFETs in amplifier circuits are typically biased into the **saturation** mode!